Analysis of AC losses due to damper windings in PMSMs

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Abstract – A passive method of noise and vibration reduction has been proposed for electrical machines, using a three-phase damper winding, called Passive Damper System (PDS). Electromagnetic noise and vibrations are effectively mitigated in a Pulse Width Modulation (PWM)-fed Permanent Magnet Synchronous Machine (PMSM) under different operating points by a PDS. However, machine efficiency is slightly affected. This paper analyzes different factors affecting machine losses in PMSMs with a PDS and discusses suitable application scenarios of PDS. The validated electrical model and experimental results are used to study machine efficiency degradation problems due to PDS. Overall, correct configurations and suitable control strategies are key to ensuring the noise reduction effect and lower additional losses due to PDSs.

Index Terms – Damper winding, PMSM, Machine efficiency, Harmonic, Noise reduction, PWM

1. INTRODUCTION

The PDS is applied in PWM-fed PMSMs to reduce electromagnetic noise and vibrations. It consists of two parts: a damper winding and external capacitors. The damper winding is installed in stator slots and superposed on the stator winding in Figure 1, where a concentrated winding machine is considered.

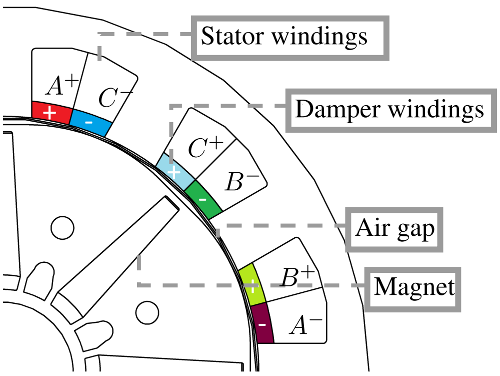

Figure 1. Schematic structure of an interior PMSM

The symbols $\pm$ and shades of colors respectively represent the direction of coil conductors in the stator and damper winding. Three phases of the damper winding are externally short-circuited via capacitors. Figure 2 shows an electrical model of a PMSM with a PDS, the damper circuits are electrically independent of the stator winding. In practice, the damper winding is also isolated from the ground. The machine or stator winding is powered by a three-phase voltage source v_s. i_s and i_a are the currents in the stator or damper winding. The capacitor, inductance and resistance together form a RLC filter in the PDS which prevents the fundamental current from flowing in the damper winding. For high frequency harmonic magnetic fields, it is easy to induce harmonic currents in the damper winding, whereas induced currents can generate the opposite magnetic field to neutralize the original harmonic magnetic fields, thus reducing electromagnetic excitation in the air-gap [1].

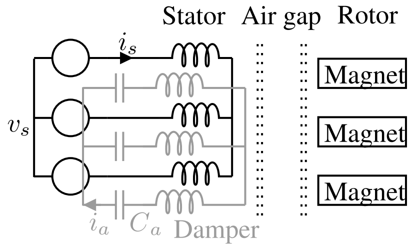

Figure 2. Electrical model of a PMSM with a three-phase passive damper system

In [2, 3], the authors successfully demonstrate that the damper system with a suitable configuration has an excellent capability of noise and vibration reduction in PMSMs and induction machines. Under different load conditions, the PDS can effectively reduce high frequency electromagnetic noise and vibrations due to PWM harmonics, thus mitigating the overall noise level, especially in low and medium-speed electrical machines.

In previous studies, the PDS configuration is optimized to improve Noise, Vibration, Harshness (NVH) reduction performance, by adjusting capacitance value. Incorrect capacitance values can cause resonance, and too large a capacitance can affect the fundamental magnetic field. The selection of capacitance value needs to consider the harmonic component as well as the operating condition. However, shortcomings of PDS should also be analyzed in depth to optimize its negative impacts on machine behavior, such as the amplification of harmonic currents in windings, machine efficiency degradation and overheating. These issues are related to harmonic components in machines, especially in both windings. Machine efficiency variation is the focus of this study, because of its importance in performance evaluation of electrical machines.

The objective of this paper is to minimize the additional losses due to PDS, by analyzing the configuration of PDS and control strategies. First, the relationship between machine losses and harmonic components is established [4]. Then the proposed electrical model (from [2]) is used to estimate the variation of harmonic components, thus predicting their impact on machine losses. Finally, experimental validation is performed to further reveal the impact of PDS on machine efficiency and noise level.

2. AC LOSSES IN A PMSM

Machine efficiency is inversely proportional to the energy consumed by the machine when the operating condition is steady.

$$\eta = \frac{P_{output}}{P_{input}} = \frac{P_{output}}{P_{output} + P_{loss}} \quad (1)$$

where η is machine efficiency, P_{input} and P_{output} are the input or output power of machine.

Machine losses P_{loss}, which are useless and wasted power, can usually be classified into three parts: copper losses, iron losses and Permanent Magnet (PM) losses [4]. In [5], the author explains the impact of PWM supplies on PMSM losses. Since the PDS is designed to reduce high frequency harmonic components, only the related machine losses are analyzed in this paper. The losses due to the fundamental component are considered unchanged.

2.1. *Copper losses*

Copper losses are related to currents flowing in copper coils. Copper losses consist of Joule losses and stray losses. Stray losses are caused by the skin effect in coil conductors and the proximity effect between conductors [6], which is proportional to Joule losses. Copper losses due to harmonic currents can be expressed:

$$P_{copper} = P_{Joule} + P_{stray} = \sum_h k_d R_{DC} (I_s^h)^2 \quad (2)$$

where R_{DC} is the DC resistance of a coil, I_s^h is the RMS value of h^{th} harmonic current flowing in the coil, and k_d is the average resistance coefficient that is the ratio of effective AC resistance versus DC resistance.

2.2. *Iron losses*

Iron losses that are related to magnetizing fields in machines can be separated into two main parts: hysteresis losses and eddy current losses [4, 5], as shown by equation (3) :

$$P_{iron} = \sum_h k_{hs} B_h^2 f_h + k_{ed} (B_h f_h)^2 + k_{ee} (B_h f_h)^{3/2} \quad (3)$$

where k_{hs} is the hysteresis loss coefficient, k_{ed} is the eddy current loss coefficient,k_{ee} is the excess eddy current coefficient, B_h is the magnitude of h^{th} flux density, f_h is the corresponding harmonic frequency. In general, B_h is considered to be proportional to magnetizing harmonic current I_μ^h [2].

2.3. *Permanent magnet losses*

Induced eddy currents generate PM losses that are trivial and negligible, compared with other losses. In [7], the author explains that PM losses are related to the square of magnetizing harmonic currents (I_μ^h).

Overall, it can be found that these losses are related to the square of currents. Copper losses are linearly proportional to the square of coil currents. Iron and PM losses depend on the magnetizing currents, especially its harmonic components.

$$P_{loss} = P_{copper}(I_s^h) + P_{iron}(I_\mu^h) + P_{PM}(I_\mu^h) \quad (4)$$

According to (4), the impact of PDS on machine losses and efficiency can be qualitatively analyzed by estimating the variation of winding current and magnetizing currents with the proposed electrical model.

3. PWM HARMONICS FROM POWER SUPPLY

High frequency electromagnetic harmonic components due to PWM power supply generally appear around integer multiples of the switching frequency (f_{swi}). The corresponding harmonic frequency can be expressed as $f_h = mf_{swi} \pm nf_e$, where f_e is the fundamental frequency, m and n are integers and do not have the same parity [8]. Their corresponding currents flow in windings and induce harmonic flux densities in the air-gap. PWM harmonics are obviously different from low frequency and fundamental components, because of their high frequency. Their impact on machine losses and vibroacoustic behavior is worth investigating. This paper focuses on the impact of PDS on high frequency PWM harmonics.

4. MODELING OF PMSMS WITH A PDS

In order to analytically study the impact of PDS in PMSMs, it is possible to model PMSMs with a PDS using electrical circuits. Due to the combination of capacitor, resistance and inductance in the damper winding, harmonic components have a high conductance in the damper circuits. However, the fundamental current can hardly flow in the PDS. Only considering high frequency harmonic components, the electrical model with the PDS can be simplified, neglecting back-electromotive force due to PM and other elements related to machine rotation. In [2], a reduced electrical model of PMSMs with a PDS is proposed in the dq frame. As shown in Figure 3, the damper winding is added in electrical circuits as a parallel branch to the magnetizing branch.

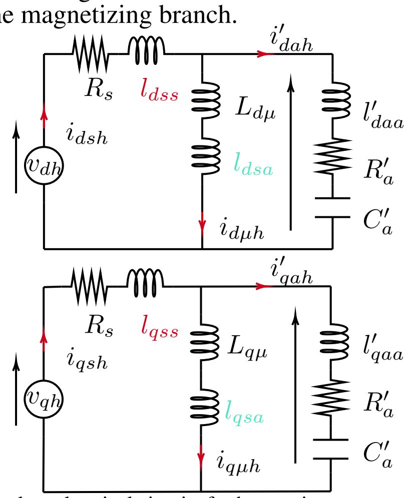

Figure 3. Equivalent electrical circuits for harmonic components on $dq-$axes

Considering the number of turns in both windings, the relative value of damper elements is expressed in electrical circuits. v_{dh}/v_{qh} is the harmonic source. R_s and R_a' are the resistances of the stator and damper winding, l_{ss} and l_{aa}' are self-leakage inductances, l_{sa} is the mutual leakage inductance between the stator and damper, L_μ is the magnetizing inductance. C_a' is the relative capacitance value brought back to the stator side. i_{sh}, $i_{\mu h}$ and i_{ah} represent stator current, magnetizing current and damper current respectively.

Using these circuits, it is also possible to estimate the variation of harmonic currents in different branches. It can be found from [2] that the magnetizing current is affected by the PDS, especially in high frequency range. According to (4), the stator and damper current contribute to copper losses and the magnetizing current is directly related to induced magnetic fields in the air-gap, which affect iron losses and PM losses. Therefore, not only vibroacoustic behavior of PMSM can be analyzed but also machine losses can be predicted with this analytical model, by estimating current variation.

5. HARMONIC CURRENT VARIATION IN A PMSM WITH A PDS

An analytical study is carried out with the proposed electrical model for a $4\mathrm{kW}$ 12-slot 8-pole interior PMSM with concentrated windings, named LEROY-SOMER permanent magnet synchronous machine (LSRPM). The copper wire diameter of the stator winding is $1\mathrm{mm}$, and that of the damper winding is $0.4\mathrm{mm}$. The stator and damper windings have the same number of turns. Other machine parameters are given in [2]. The variation of harmonic current can be analyzed with Frequency-Domain Equivalent Electrical Circuits (FDEECs), where all electrical elements are considered vectors. Different harmonics are injected as voltage sources.

In this investigation about PWM harmonics analyses, the following assumptions are considered:

- Machine rotation is not taken into account;
- Initial phase of harmonic voltages is zero for different frequencies;
- RMS value of harmonic voltage source is constant at $V_h^{RMS} = 40\mathrm{V}$;
- Permanent magnets are considered as a vacuum or air region;
- Magnetic permeance on the d-axis is considered to be the same as that on the q-axis in the studied machine;
- Skin effect is negligible in the electrical model because the skin depth of PWM harmonics is thicker than copper wire radius, and AC resistance is equal to DC resistance;

The electrical model can analyze the evolution of currents versus various variables. In this study, the capacitance value C_a and harmonic frequency f_h can be considered two varying factors. The capacitance value C_a varies from 0 to 10μF. Furthermore, the harmonic frequencies correspond to the audible range from 20Hz to 20000Hz. Figure 4 and Figure 5 show the evolution of harmonic currents versus capacitance value and frequency respectively in each circuit branch on the $d-$axis.

In Figure 4, the magnetizing currents first increase to the resonance peaks, and then they tend to a constant value that is lower than their initial value. When the capacitance value is great enough, the magnetizing current evolution is almost unchanged. The resonance peak shifts to the left with the increase of harmonic frequency, and its width becomes narrower. Therefore, the PDS with suitable capacitance values can possibly reduce harmonic components in the magnetizing branch and equivalently mitigate electromagnetic excitation in the air-gap, which is the principle of the passive noise and vibration reduction method using a PDS. Harmonic currents have a similar trend in the stator and damper branches, but their curve converges to a final value that is higher than their initial value. In other words, harmonic components are unavoidably amplified in stator and damper windings. A relatively greater capacitance value has a beneficial effect to minimize the current amplification due to the PDS. When the capacitance value is large enough, this beneficial effect becomes weaker and negligible. Furthermore, in [2], the author points out that an excessive capacitance value can make the fundamental magnetic field affected in the air-gap, which is undesirable. Therefore, a suitable capacitance value should help the PDS not only to reduce high frequency harmonic components in magnetizing currents but also to limit the current amplification in the stator and damper circuits.

In Figure 5, the current evolution versus harmonic frequency with several given capacitance values is shown. The initial system is considered reference without PDS. In a PMSM with a PDS, harmonic currents are obviously impacted. First, in the magnetizing branch, harmonic currents are obviously amplified by electrical resonances, compared to the reference curve. Then, they decrease and are below the reference curve. Therefore it is revealed that the electrical resonance should be avoided and a higher harmonic frequency component is more easily reduced by the PDS in magnetizing currents. In the stator branch, harmonic currents sharply decrease to the minimum, and then they are affected by the electrical resonance, finally, the currents tend to a constant value but are always above the reference curve. In the damper branch, the current evolution rises up to the resonance peak, and then it gradually decreases to a constant value that is greater than the initial one. In the initial system, the damper current is considered zero. From this investigation, it can be found that a higher harmonic frequency is favorable for PDS operation, especially for reducing current amplification in the stator and damper windings caused by the PDS and mitigating harmonic components in magnetizing currents.

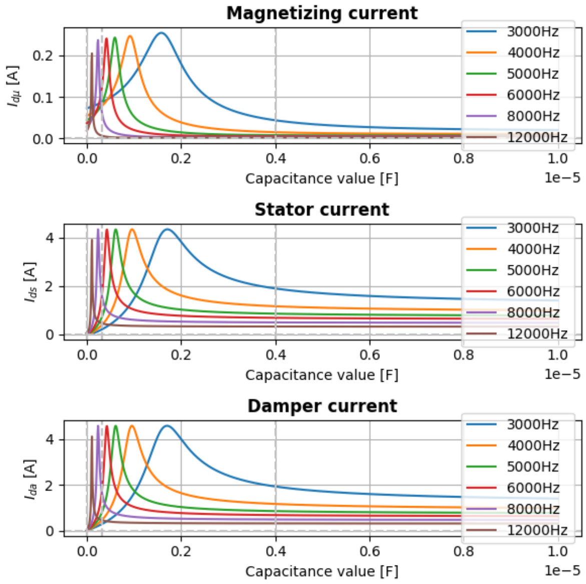

Figure 4. Harmonic currents versus capacitance value under different frequencies in a PMSM with a PDS

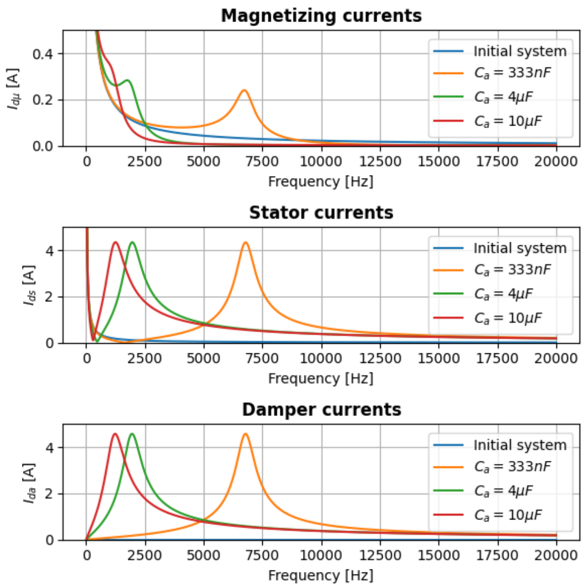

Figure 5. Harmonic currents versus frequency in a PMSM with a PDS and different capacitance values

6. IMPACT OF PDS ON AC LOSSES DUE TO PWM HARMONICS

From the above section, the current variation has been discussed. Combining the equation (4), this section analyzes the impact of PDS on AC losses related to PWM harmonics in a PMSM. Different types of losses can separately be discussed as follows:

- **Iron and PM losses**: These two losses are related to the magnetizing current, especially harmonic components.

From Figure 4 and Figure 5, it can be deduced that the PDS with suitable capacitance values effectively reduces iron and PM losses by significantly mitigating harmonic components in the magnetizing current. The harmonic components are equivalently reduced in the air-gap flux density. The noise and vibration reduction also benefit from this phenomenon [3, 1]. From (4), iron and PM losses are also related to these harmonic flux densities. Therefore, the PDS has a positive and consistent effect on optimizing NVH and reducing iron and PM losses due to high frequency harmonics.

- **Joule losses**: Joules losses or copper losses are mainly contributed by Joule effect in the stator and damper winding wires, which is related to winding currents. Figure 4 and Figure 5 reveal that additional Joule losses due to the PDS are unavoidable in a PMSM, because of harmonic current amplification in the stator and damper winding. Larger capacitance values are beneficial for further mitigating harmonic current amplification to reduce the corresponding Joule losses in both windings. However, this beneficial effect diminishes as the capacitance value increases. When the capacitance value is large enough, continuing to increase the capacitance value does not further reduce the additional Joule losses.

It can be summarized that additional Joule losses should further be optimized to minimize the impact of PDS on machine efficiency. However, it is difficult to effectively reduce Joule losses due to PWM harmonics in windings by continuing to increase the capacitance value in an already reasonably configured PDS. An alternative is to explore the application scenarios suitable for PDSs. Exactly, under higher harmonic frequencies, the harmonic components can effectively be reduced in the magnetizing current, and the current amplification can also be limited in the stator and damper winding. This situation seems to be favorable for the PMSM with a PDS to further reduce additional Joule losses. According to (3), iron losses are positively correlated to harmonic frequencies and the square of air-gap flux densities. Due to the PDS, harmonic flux densities are substantially reduced. The higher harmonic frequency may cause a change in iron losses, but such a change is negligible compared to the damping effect of the PDS on harmonic flux densities. Iron losses and PM losses can still be reduced thanks to the PDS. Therefore, the variation of machine efficiency at different PWM harmonics is worth to be practically studied for PMSMs with a PDS.

7. EXPERIMENTAL VALIDATION

In [1], a test bench has been introduced. This test bench is developed for characterizing the LSRPM with a PDS under different operating conditions. The studied machine is coupled with a DC machine (DCM) and is powered by a converter SEMITEACH. With the help of dSpace® (MicroLabBox®), a close-loop Field-Oriented Control (FOC) is realized. To precisely measure the vibroacoustic performance of the studied machine, a semi-anechoic chamber isolates the machines from the operation desk.

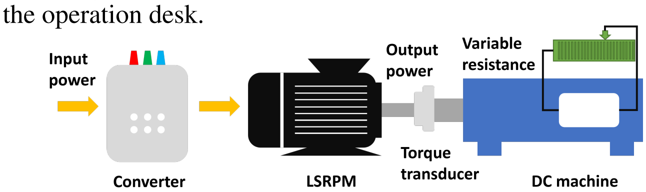

Figure 6. Power measurement in test bench

Machine efficiency is considered the ratio of output mechanical power to input power provided by the grid in this study as shown in 6. The output torque and rotation speed are measured to estimate the output power. Commutation losses in the converter are also taken into account. Iron losses and PM losses are difficult to directly and separately measure in experiments, but it is possible to qualitatively evaluate the impact of PDS by observing the variation of harmonic components in the air-gap magnetic field. The air-gap magnetic field can be measured by a single-turn search coil that is installed in stator slots following one of three phases. In [1, 2], the importance of applying suitable capacitance values has been demonstrated. With FFT spectrum analysis, it is confirmed that the PDS can effectively and significantly reduce PWM harmonic components in the air-gap magnetic field in practice. This finding indirectly confirms the beneficial effect of PDS with a correct configuration on iron and PM losses. However, it is also found that the overall machine efficiency has decreased and the losses have increased. It is necessary to comprehensively study the impact of PDS on machine losses under different operating conditions.

Several operating conditions are tested as shown in Figure 7. The operating map can intuitively show the state of LSRPM under different load conditions. The abscissa represents the rotation speed in RPM, the ordinate represents the applied load case. Exactly, it corresponds to the variable resistance on DCM. It is impossible to keep the same output torque and power in each case while varying rotation speed. It is noted that in no-external load conditions, the machine has to output mechanical power to keep the rotation speed, overcoming friction and cogging torque, so the couple transducer can measure a weak output mechanical couple as shown in Figure 7.

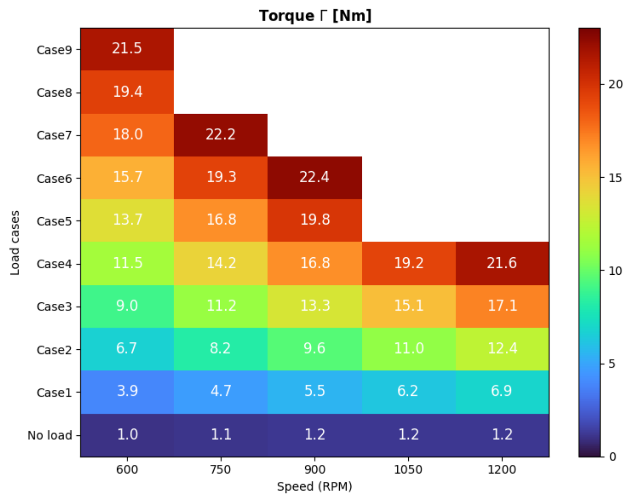

Figure 7. Operating map of LSRPM

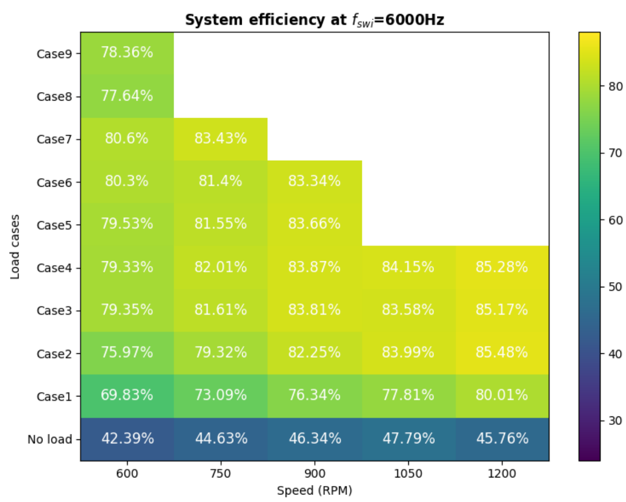

Figure 8. Efficiency map of LSRPM at $f_{swi} = 6000$Hz

Figure 8 shows efficiency map of LSRPM without a PDS at switching frequency $f_{swi} = 6000$Hz. In experiments, f_{swi}

varies from 3000Hz to 10000Hz, efficiency map of LSRPM is almost identical, so the variation of commutation losses in the inverter is negligible in this study. Machine efficiency is improved under heavier load conditions because machine losses become less important compared to output mechanical power.

7.1. *Impact of PDS on machine efficiency*

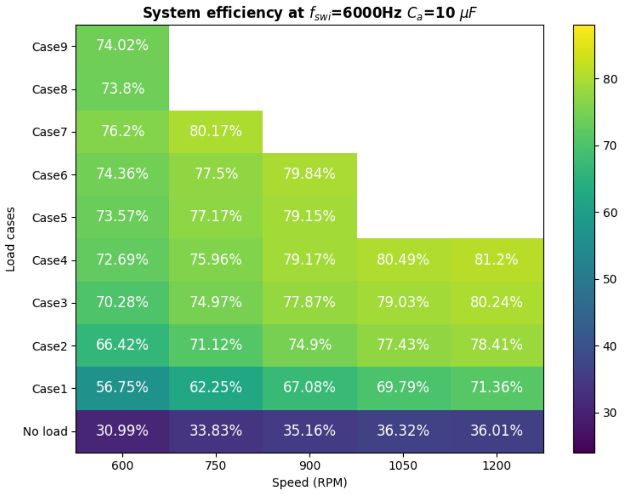

Figure 9. Efficiency map of LSRPM at $f_{swi} = 6000\text{Hz}$ and $C_a = 10\mu\text{F}$

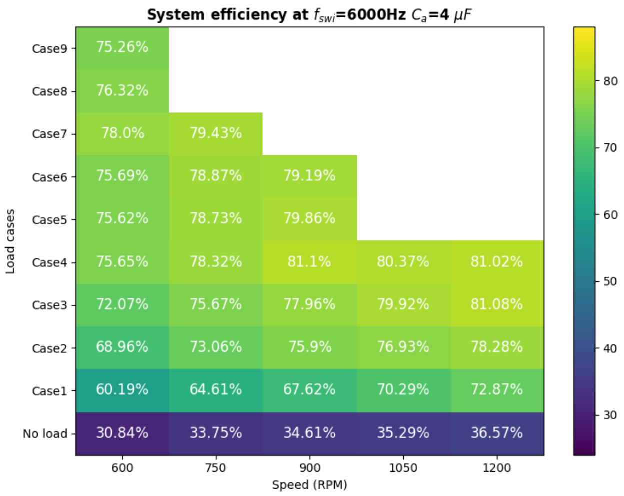

Figure 10. Efficiency map of LSRPM at $f_{swi} = 6000\text{Hz}$ and $C_a = 4\mu\text{F}$

With the same switching frequency $f_{swi} = 6000\text{Hz}$, two efficiency maps of LSRPM with a PDS at $C_a = 10\mu\text{F}$ and $C_a = 4\mu\text{F}$ are plotted in Figure 9 and Figure 10. It can be found that the PDS significantly affects machine efficiency. The efficiency varies from 31% to 81% instead of from 42% to 85%, depending on operating conditions. Efficiency degradation is relatively reduced at heavier load conditions. However, at low and no-load conditions, the negative impact of PDS on machine efficiency is significant. Although iron losses and PM losses related to PWM harmonics are theoretically reduced, total machine losses are still increased. It can be deduced that the optimization of additional Joule losses due to harmonic current amplification in windings is an important solution to improve machine efficiency. Furthermore, $C_a = 4\mu\text{F}$ and 10µF do not make an obvious difference on machine efficiency in the LSRPM with a PDS, machine efficiency degradation identically exists. In previous studies, both capacitance values are considered suitable to reduce electromagnetic noise and harmonic-induced vibrations. The relatively greater capacitance value (10µF) does not have a significantly better effect, compared to 4µF. This finding is also in agreement with the analytical study about predicting variation of machine losses with Figure 4 and Figure 5. With a properly configured PDS, the adjustment of capacitance values hardly improves machine efficiency or reduces additional losses due to the PDS.

7.2. *Impact of harmonic frequency on machine efficiency*

Adjusting the switching frequency f_{swi} can change the frequency of PWM harmonic components. Keeping the same operating condition and modulation index, the amplitude of harmonic voltages is unchanged while changing the switching frequency f_{swi}.

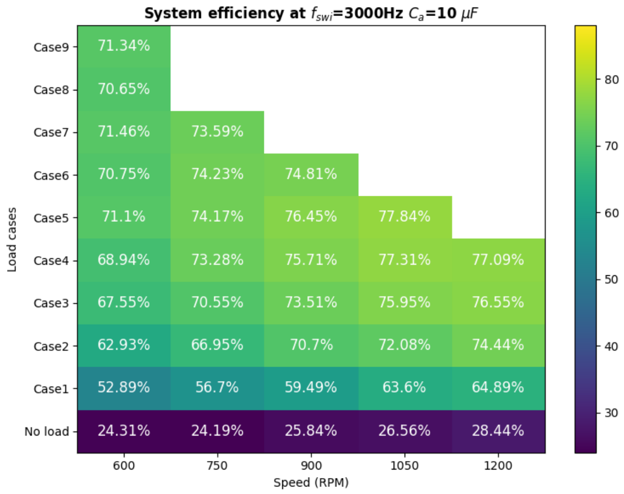

Figure 11. Efficiency map of LSRPM at $f_{swi} = 3000\text{Hz}$ and $C_a = 10\mu\text{F}$

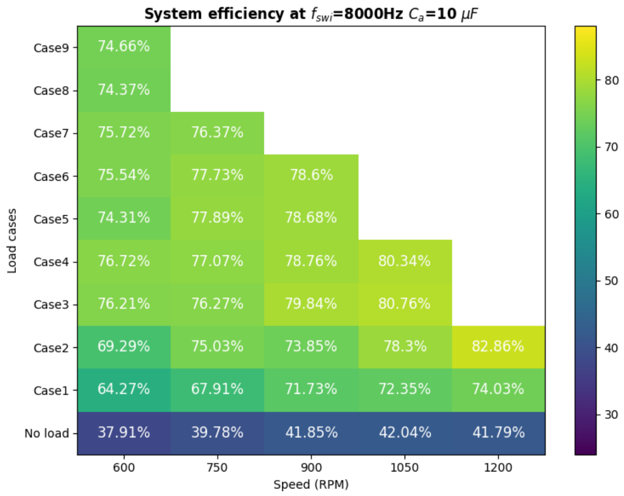

Figure 12. Efficiency map of LSRPM at $f_{swi} = 8000\text{Hz}$ and $C_a = 10\mu\text{F}$

At $C_a = 10\mu\text{F}$, two other switching frequencies (3000Hz and 8000Hz) are tested in experiments to study the impact of harmonic frequency on the efficiency of LSRPM with a PDS. Figure 11 and Figure 12 show two efficiency maps with two switching frequencies. It can be found that $f_{swi} = 3000\text{Hz}$, globally, machine efficiency is further mitigated at each operating condition, compared to the case at $f_{swi} = 6000\text{Hz}$ and the initial machine. This finding is consistent with the theoretical analysis. With $f_{swi} = 3000\text{Hz}$, the corresponding harmonic frequencies are lower, so the current amplification and additional Joule losses are greater. The impact of PDS on machine efficiency is more obvious. On the contrary, at $f_{swi} = 8000\text{Hz}$, machine efficiency is truly improved at low and no-load conditions thanks to higher harmonic frequencies. Under heavier load conditions, the beneficial effect due to the higher switching frequency becomes less important, compared to total output power, so the difference is trivial and negligible between cases with $f_{swi} = 6000\text{Hz}$ and the case with $f_{swi} =$

8000Hz. In summary, under operating conditions with higher harmonic frequencies, machine efficiency degradation due to the PDS is less obvious, especially at low and no-load conditions.

8. NOISE LEVEL EVALUATION

Overall sound pressure level is measured for each study case, it can be found the noise level is truly reduced by the PDS in three cases with different f_{swi}, compared to the initial machine without PDS. Figure 13, Figure 14 and Figure 15 show the noise level variation in dB. According to previous studies [1], PWM electromagnetic high frequency noise dominates the noise level in this low-speed PMSM. Exactly, the PDS reduced electromagnetic noise, thus reducing the overall noise level. It can be deduced that high frequency harmonic components are effectively reduced in the air-gap magnetic field under three study cases. It is important to note that the noise reduction effect seems to be significantly different in these three cases. However, this finding does not judge the difference in noise reduction performance of PDS at different PWM harmonics. This is because noise signatures due to different harmonics are inconsistent and are different even in the initial machine without PDS. When the electromagnetic noise is effectively reduced, the overall noise level of the studied machine is dominated by mechanical noise. Therefore, this study only qualitatively demonstrates that the PDS can effectively improve the acoustic characteristics and reduce electromagnetic harmonics in all three cases.

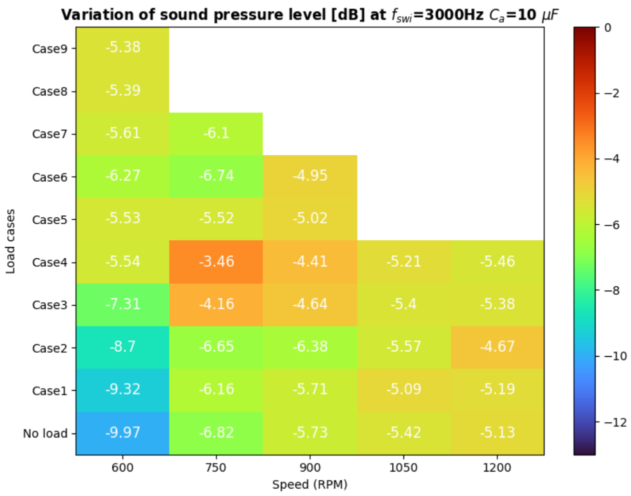

Figure 13. Noise level variation of LSRPM at $f_{swi} = 3000$Hz and $C_a = 10\mu$F

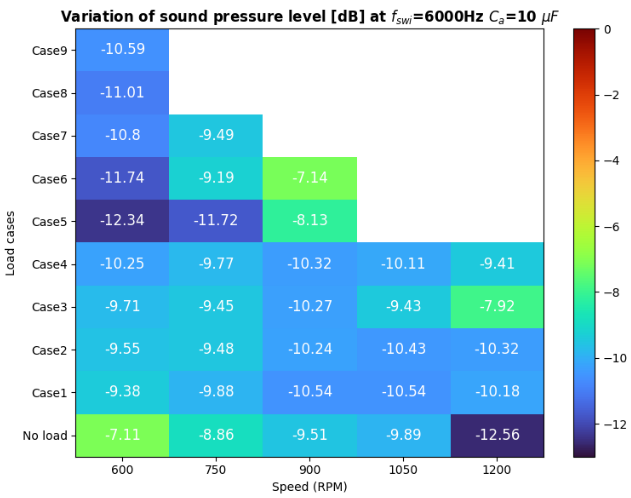

Figure 14. Noise level variation of LSRPM at $f_{swi} = 6000$Hz and $C_a = 10\mu$F

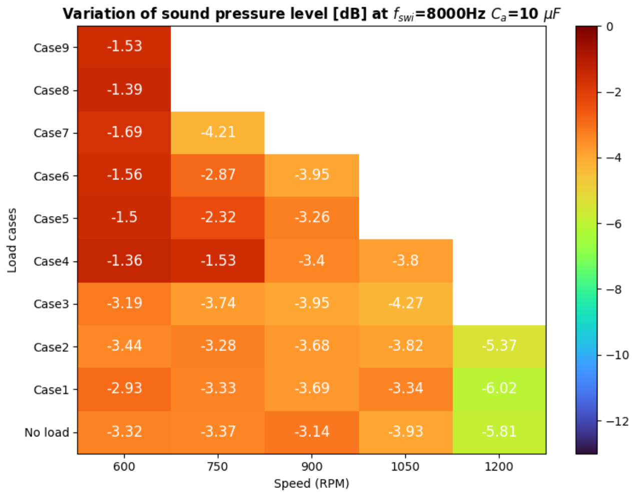

Figure 15. Noise level variation of LSRPM at $f_{swi} = 8000$Hz and $C_a = 10\mu$F

9. CONCLUSIONS

In this paper, machine losses are analyzed, and the relationship between machine losses and harmonic components is presented. Using the proposed electrical model, the variation of harmonic currents in different branches is estimated, and its impact on machine losses is qualitatively explained. Experimental validation is performed to evaluate the impact of PDS on machine efficiency and overall noise level. Combined with the previous studies, it can be concluded that the suitable capacitance value is the key to achieving noise reduction and are also favorable for reducing iron and PM losses. The PDS is more suitable for low and medium-speed PMSM with heavier loads and higher harmonic frequencies scenarios, where it can effectively mitigate the overall noise level and can further reduce its negative impact, especially on machine efficiency.

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