abc-Modeling of Permanent Magnet Machines using N-D Lookup Tables: a Finite Element Validation

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ABSTRACT – The lumped-parameter nonlinear abc-model using $i(\lambda)$ N-dimensional lookup tables can naturally include geometry- and saturation-induced space harmonics. It is as accurate as the finite element model while providing a significant speedup. We validate here this model for permanent magnet synchronous machines by extensive comparison with the circuit-coupled finite element model. Three different types of machines (interior permanent magnet machine, surface mounted permanent magnet machine, flux switching permanent magnet machine) are considered, in both steady and transient state, for balanced and unbalanced operation.

KEYWORDS – abc-model, phase-domain model, space harmonics, saturation, modeling, simulation, validation.

1. INTRODUCTION

For the simulation of electrical machines and their coupling with external circuits, a fast and accurate machine model is always desirable. On the one hand, the classical lumped-parameter dq-model is efficient. However, it is established on a number of approximations that are not always applicable. On the other hand, the finite element (FE) model gives accurate results, but it is computationally expensive. Therefore, the lumped-parameter abc-model (or phase-domain model) has progressively re-gained attention [1–6]. The difficulty of the nonlinear abc-model comes from the fact that its parameters are functions of the rotor angle and the windings currents. To deal with these parameters without making any approximation, the authors proposed a novel implementation using current-flux linkage $i(\lambda)$ and electromagnetic torque N-dimensional lookup tables (N-D LUT) [7]. It is as accurate as the FE model because it includes the space harmonics linked to both the magnetic circuit geometry and the (cross-)saturation. But, it is about two orders of magnitude faster. Besides, its parameters are physical values, its derivation is straightforward, and its implementation is independent of the machine type, geometry or materials. Earlier, we validated this model for an electrically-excited synchronous machine by comparison with the circuit-coupled FE model.

In this article, we extend the validation to permanent magnet synchronous machines (PMSM). This is of interest because the use of permanent magnets and peculiar topologies usually require a special care when building a lumped-parameter model, and calculating its parameters. First, we recall the abc-model hypotheses and equations. This is the occasion to clarify the limitations of inductance-based abc-models for modeling PMSMs. Then we underline the capacity of the $i(\lambda)$-based abc-model in modeling PMSMs. The model is validated for three different types of PMSMs by extensive comparison with the circuit-coupled FE model. Finally, we discuss some possible applications of the abc-model.

2. ABC-MODEL EQUATIONS

We use matrix notation in motor convention, meaning that positive currents enter the machine terminals. In this article, we consider specifically 3-phase PMSMs. The nonlinear abc-model is,

\begin{equation}
\mathbf{v} = \mathbf{R}\mathbf{i} + \frac{d\lambda(\theta, \mathbf{i})}{dt}
\end{equation}

\begin{equation}
J^2\frac{d^2\theta}{dt^2} = T_m + T_e(\theta, \mathbf{i}) + T_{damp}
\end{equation}

where $\mathbf{v}$, $\mathbf{i}$, and $\lambda$ are the instantaneous voltage, current and flux linkage of the stator windings, respectively. $\theta$ is the rotor mechanical angle. $\mathbf{R}$ is the resistance matrix of the windings. $J$ is the rotor moment of inertia. $T_m$, $T_e$ and $T_{damp}$ are the mechanical torque, the electromagnetic torque and the damping torque, respectively. $\lambda$ and $T_e$ are nonlinear functions of rotor position and winding currents.

2.1. Inductance-based abc-model

Implementations of the abc-model is generally realized by extending the inductance concept to the nonlinear case. Mohammedi et al. [2] derived a nonlinear abc-model using the current $\mathbf{i}$ as state variable. By separating the flux linkage contributed by the stator winding $A_\lambda$ from the one contributed by the permanent magnets $A_m$, they obtained,

\begin{equation}
\frac{d\mathbf{L}}{dt} = L^a(\theta, \mathbf{i})^{-1} \left[ \mathbf{v} - \mathbf{R}\mathbf{i} - \omega \frac{\partial L^a(\theta, \mathbf{i})}{\partial \theta} \mathbf{i} - \mathbf{e}_s(\theta, \mathbf{i}) \right]
\end{equation}

where $L^a_{s,jk} = \frac{\partial A_{s,jk}}{\partial \mathbf{i}}$, $L^a_{s,jk} = \frac{\partial A_s}{\partial \mathbf{i}}$, $\omega$ is the rotor mechanical speed and $\mathbf{e}_s$ is called "back-emf". Poltschak et al. [5] derived a different nonlinear abc-model using the current $\mathbf{i}$ as state variable but without resorting to flux linkage separation,

\begin{equation}
\frac{d\mathbf{L}}{dt} = L^a(\theta, \mathbf{i})^{-1} \left[ \mathbf{v} - \mathbf{R}\mathbf{i} - \mathbf{e}(\theta, \mathbf{i}) \right]
\end{equation}

where $L^a_{s,jk} = \frac{\partial A_{s,jk}}{\partial \mathbf{i}}$ and $\mathbf{e}$ is called "generalized back-emf". Another possibility is to use the flux linkage $\lambda$ as state variable. Extending the derivation of [8] to PMSMs, one obtains,
Fig. 1. Permanent magnet machine models used for FE validation.

\[
\frac{d\lambda}{dt} = v - RL^a(\theta, i)^{-1} [\lambda - \lambda_m(\theta, i)]
\]

where \( L_{jk}^a = \frac{\lambda_j}{i_k} \). From the above expressions, the limitation of inductance-based abc-modeling is clear. First, the necessary distinction between apparent inductance \( L^a \) and differential inductance \( L^d \) is error-prone during both parameters calculation and implementation. Secondly, \( L^d \) contains partial derivatives of the flux linkages with respect to the currents. The numerical derivation increases both computation time and numerical errors. Thirdly, it is not necessarily clear how to calculate the "(generalized) back-emf". Finally, a 3-phase PMSM requires the specification of at least one \( 3 \times 3 \) matrix and one \( 3 \times 1 \) matrix, which coefficients are each a function of the rotation angle \( \theta \) and the 3 currents. Handling such a volume of data further increases the complexity of the model.

### 2.2. \( i(\lambda) \)-based abc-model

As proposed in [7], we use the flux linkage as state variable, and the current-flux linkage \( i(\lambda) \) instead of the inductance. The nonlinear abc-model is then simply,

\[
\frac{d\lambda}{dt} = v - Ri(\theta, \lambda)
\]

The existence of \( i(\theta, \lambda) \) is guaranteed by the fact that a change of variable can be done between \( i \) and \( \lambda \) under the hypothesis that (1) the losses of the coupling field (eddy current, hysteresis, or dielectric losses) are negligible [9], and (2) that the windings are not perfectly coupled [7]. The advantages of this model are that the variables meaning is clear and that, a 3-phase PMSM requires only the specification of one \( 3 \times 1 \) matrix, which coefficients are each a function of the rotation angle \( \theta \) and the 3 currents. One general way to define \( i(\theta, \lambda) \) without making any further approximation is to use three 4-D lookup tables (LUT). The integration of Eq.6 is straightforward once the LUTs \( i(\theta, \lambda) \) have been specified. As \( i(\theta, \lambda) \) is not directly available, it must be obtained from \( \lambda(\theta, i) \). We showed in [7] that this can be done using inversions and 3-D interpolations. The implementation of Eq.6 in Matlab/Simulink has been discussed in [7].

### 2.3. Electromagnetic torque

In previous works [2, 5, 6], the electromagnetic torque is generally obtained by separation of the synchronous/reluctance torque from the cogging torque. Once more, this introduces difficulties for both parameters calculation and implementation. Therefore, we use directly the electromagnetic torque [7]. It can be obtained by FE calculation using the Maxwell stress tensor at the same time as the \( \lambda(\theta, i) \) characteristic, and stored in one 4-D LUT.
3. ABC-MODEL VALIDATION

In this section, we validate the $i(\lambda)$-based nonlinear abc-model using N-D LUTs for permanent magnet synchronous machines (Eq.6) by comparison with the circuit-coupled finite element model. Three different types of machines are considered: interior permanent magnet machine (IPM), surface mounted permanent magnet machine (SPM) and flux switching permanent magnet machine (FSPM). The machines are 3-phase PMSM, and therefore three 4-D LUTs are used for the electric equation. One 4-D LUT is used for the electromagnetic torque equation. The LUTs are calculated by static 2-D finite element analysis.

Because the abc-model deals with the electrical part of the machine, we consider generator operation at fixed speed. The armature windings are connected to a three phase resistive load. We do not use an inductive/capacitive load to ensure that the current/voltage waveforms harmonics are due only to the space harmonics.

A complete validation should (1) cover a wide range of stator currents and (2) include various transient operations. The former is obtained for each machine by varying the rotor speed at constant load. The latter is done, to keep the article concise, by simulating one different transient for each machine. Even so, this allows us to evaluate both balanced and unbalanced operation, with star point floating or grounded.

3.1. Interior PMSM

We consider the interior PMSM shown in Fig.1(a), adapted from [10]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{align*}
\theta & \in \{0 : 1.875 : 90\} \text{ meca deg} \\
i_{abc} & \in \{0.5, 1, 1.3\}^3 \times \pm 10 \text{ A}
\end{align*}$$  (7)

Fig.2(a) shows the steady-state armature current as a function of the rotor speed. To illustrate the ability of the abc-model in modeling unbalanced transients with star point floating, we simulate a single-phase-to-ground fault. Initially the machine is in steady-state operation ($\omega_m = 1040$ rpm, $R_{load} = 12.5 \Omega$). At $t_0$, phase $a$ is suddenly connected to the ground during 1 ms. Fig.2(b) shows the armature currents, voltage and the electromagnetic torque during the transient.

3.2. Surface mounted PMSM

We consider the surface mounted PMSM shown in Fig.1(b), adapted from [11]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{align*}
\theta & \in \{0 : 3.75 : 180\} \text{ meca deg} \\
i_{abc} & \in \{0.5, 1, 1.3\}^3 \times \pm 5 \text{ A}
\end{align*}$$  (8)

Fig.3(a) shows the steady-state armature current as a function of the rotor speed. To illustrate the ability of the abc-model in modeling unbalanced transients with star point grounded, we simulate a phase-to-phase fault. Initially the machine is in steady-state operation ($\omega_m = 3000$ rpm, $R_{load} = 33.75 \Omega$). At $t_0$, phases $a$ and $b$ are suddenly connected during 1 ms. Fig.3(b) shows the armature currents, voltage and the electromagnetic torque during the transient.

3.3. Flux switching PMSM

We consider the flux switching PMSM shown in Fig.1(c), adapted from [12]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{align*}
\theta & \in \{0 : 1 : 36\} \text{ meca deg} \\
i_{abc} & \in \{0.5, 1, 1.3\}^3 \times \pm 20 \text{ A}
\end{align*}$$  (9)

Fig.4(a) shows the steady-state armature current as a function of the rotor speed. To show the ability of the abc-model in modeling reluctance torque ripples and balanced transients, we simulate a sudden R load switching. Initially the machine is in steady-state operation at no-load ($\omega_m = 1000$ rpm, $R_{load} = \infty$). At $t_0$, the 3-phase resistive load ($R_{load} = 7.5 \Omega$) is suddenly connected. Fig.4(b) shows the armature currents, voltage and the electromagnetic torque during the transient.
3.4. Discussion

The nonlinear abc-model using \(i(\lambda)\) N-D LUTs and the circuit-coupled finite element model are in very good agreement for all machines and all operating conditions. The slight torque difference between the two models is attributed to the difficulty of torque calculation with the FE model; both the static FE model used to fill up the LUTs and the transient FE model. Note that the abc-model is able to correctly predict operation for LUT out of the range (Figs. 2(a), 3(a) and 3(b)), using LUT cubic spline extrapolation for the currents and LUT linear extrapolation for the electromagnetic torque. This validates the ability of the abc-model in modeling geometry- and saturation-induced space harmonics, during steady and transient state, for balanced and unbalanced operation, with star point floating or grounded. Comparison of the models in terms of computation time has been discussed in [7].
converter (not its model) [16]. This allows to test equipments before on-site installation, to tune controllers for best performance and to shorten development cycles. In this context, specialized hardware-in-the-loop real time simulation tools, such as RTDS® or eDRIVEsim®, have been developed. They are able to simulate PM synchronous machines in real-time (< 1μs time step) with high accuracy. The proposed abc-model is a good candidate for such implementations because it uses fewer lookup tables than inductance-based models [4].

4.3 Condition monitoring and fault diagnosis

Machine condition monitoring and fault diagnosis are important for the reliable operation of power systems. A fast and accurate machine model can be used to: obtain machine fault signatures contained in operating signals, evaluate on-line fault diagnosis strategies, or investigate the impact of machine faults on external circuits (excitation and loads). Besides, internal faults in synchronous machines can be simulated by minor modifications of the abc-model [3, 17, 18].

4.4 Noise analysis

Because it includes geometry- and saturation-induced space harmonics, the abc-model can be used for modeling, assessing, and minimizing acoustic noise in electric machines and drives. In reference [19], Boesing et al. calculate the electromagnetic forces acting in the air-gap of the machine using a similar model. In combination with structural vibration or radiation models, one could then estimate the acoustic noise of the machine under arbitrary operating conditions.

5. Conclusion

We validated the ability of the lumped-parameter nonlinear abc-model using i(λ) N-D lookup tables in modeling permanent magnet machines, including IPEM, SPM, and FSPM machines. The model estimates with the same accuracy as the circuit-coupled finite element model both geometry- and saturation-induced space harmonics, during steady and transient state, for balanced and unbalanced operation, with star point floating or grounded. Such a model is suited for applications requiring both high accuracy and high computing speed, such as simulation of integrated power system, hardware-in-the-loop real time simulation, machine condition monitoring and fault diagnosis, or noise analysis.

6. References